## Conclusions

Present experiments, extending earlier data in constant pressure flows, show that the wedge angle increases slowly with Reynolds number. A favorable pressure gradient, presumably because of its stabilizing effect, inhibits the growth of turbulent spots, and, in general, results in a nonlinear turbulent wedge. However, as soon as the pressure gradient diminishes to the point where the flow becomes supercritical, spot growth picks up rapidly and the associated turbulent wedge becomes linear.

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# Injection into a Turbulent Boundary Layer Through Different Porous Surfaces

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## Nomenclature

| | |
|---|---|
| $C_f$ | = skin friction coefficient |
| $Re_L$ | = Reynolds number based on length |
| $U$ | = local mean velocity |
| $U_e, UE$ | = edge velocity |
| $U_*$ | $=\sqrt{\tau_w/\rho}$ |
| $U^+$ | $= U/U_*$ |
| $\sqrt{\overline{u'^2}}/U_e$; $U'/UE$ | = axial turbulence intensity |
| $\sqrt{\overline{v'^2}}/U_e$; $V'/UE$ | = normal turbulence intensity |
| $\overline{U'V'}/U_*^2$ | = Reynolds stress |
| $V_0$ | = injection velocity |
| $V_0^+$ | $= V_0/U_*$ |
| $Y$ | = vertical distance from wall |
| $Y^+$ | $= YU_*/\nu$ |
| $\tau_w$ | = wall shear |
| $\rho$ | = density |
| $\nu$ | = laminar kinematic viscosity |

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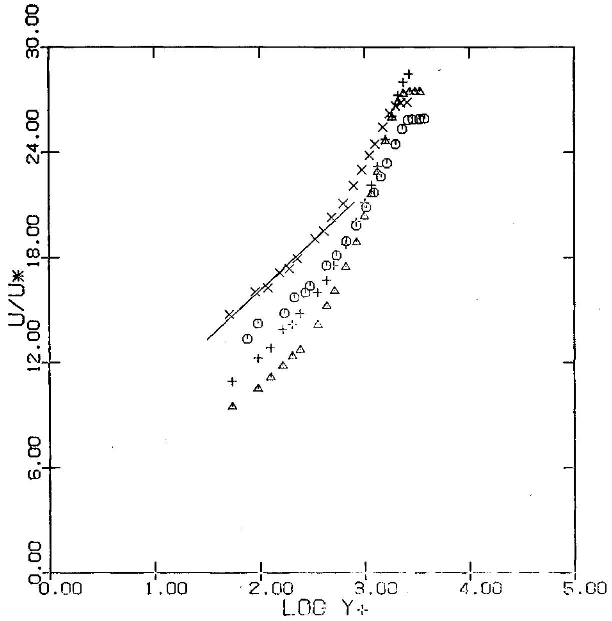


**Fig. 1 Law of the wall profiles. Perforated titanium wall at $Re_L = 4.96\times10^6$: ○, $V_0^+=0.14$; △, $V_0^+=0.19$. Porous, sintered wall at $Re_L=5.76\times10^6$ (Ref. 9): +, $V_0^+=0.10$. Smooth, solid wall at $Re_L=4.96\times10^6$: ×. Calculated from Clauser for a smooth, solid wall:———.**

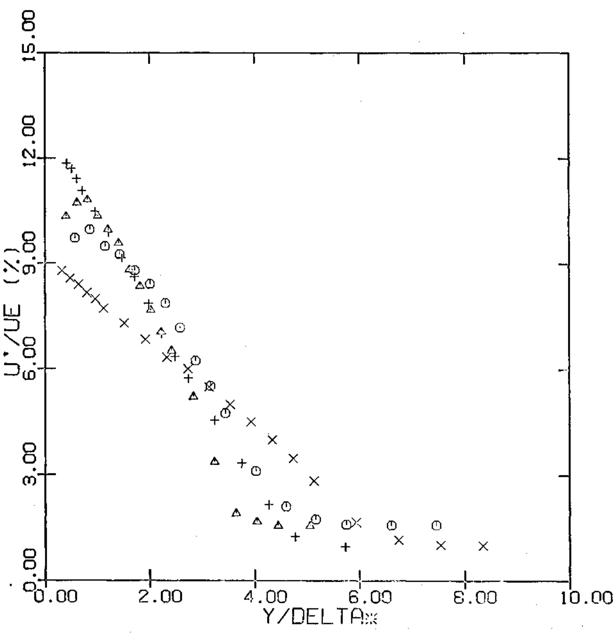


**Fig. 2 Axial turbulence intensity profiles. Perforated titanium wall at $Re_L=4.96\times10^6$: ○, $V_0^+=0.14$; △, $V_0^+=0.19$. Porous, sintered wall at $Re_L=5.76\times10^6$ (Ref. 9): +, $V_0^+=0.10$. Smooth, solid wall at $Re_L=4.96\times10^6$ (Ref. 9): ×.**

## Introduction

ON the subject of turbulent boundary layer flows over permeable surfaces with injection or suction, all but the most recent work has been surveyed in Refs. 1-5. In addition, several studies have been devoted to extending the law of the wall to a form applicable to turbulent flows with injection or suction.[6-9] Practical applications include: transpiration cooling, drag reduction on airfoils or other aero/hydrodynamic surfaces, boundary layer control, and chemical processing, as well as others.

A wide variety of porous surfaces have been employed in the experimental studies including sintered metal, layered screening, and even an array of tiny metal balls. There has been no previous study of the relative performance of different porous surfaces. A porous material with less roughness than the sintered or screening materials would be attractive. For this purpose, a titanium sheet (0.625 mm thick) with 0.15-mm-diam holes drilled in a 0.625 mm center to center pattern using an electron beam was selected as a candidate. The results are to be compared with those of Ref. 9 in which a porous, sintered wall and a smooth, solid wall were employed. The titanium has greater strength per unit weight and a smoother surface; however, the sintered metal used in Ref. 9 has a more uniformly porous surface which could make it more effective in reducing the skin friction in the case of injection.

The current investigation was conducted in the same apparatus under similar testing conditions as previous work [9-11] on a smooth, solid wall and a porous, sintered wall. The results for the smooth, solid wall reported in Ref. 9 were in excellent agreement with well established results and lent credibility to the testing apparatus and procedure. Direct measurements were taken of the wall shear utilizing a floating element balance, the velocity distribution using a Pitot rake, and axial and normal turbulence intensities and Reynolds stress profiles utilizing hot wire anemometry. The measurements were obtained at one axial location at a nominal speed of 45.1 m/s corresponding to a $Re_L$ of 4.96 × $10^6$.

## Results

The excellent agreement between Clauser's (1956) law of the wall and the experimental results reported in Refs. 9-11 for the smooth, solid wall is shown in Fig. 1. In addition, the effects of injection on the wall law are presented in Fig. 1. A downward shift of $\Delta U^+ \simeq 1.50$ in the logarithmic region is observed for the perforated titanium wall at $V_0/U_* = 0.14$. This is a slight decrease of $\Delta U^+$ compared to the no injection case. Increasing the injection velocity to $V_0^+ = 0.19$ results in a downward shift of $\Delta U^+ \simeq 4.50$. This is greater than the no injection case. A downward shift of $\Delta U^+ \simeq 3.0$ is observed for the porous, sintered metal wall at $V_0^+ = 0.10$. These results indicate that the wall law is a quite complicated function of the injection rate and the magnitude and pattern of the roughness and the porosity of the porous material employed.

Figure 2 is a comparison of the axial turbulence intensity of the porous materials and the smooth, solid wall. Injection increases the axial turbulence near the wall and causes the

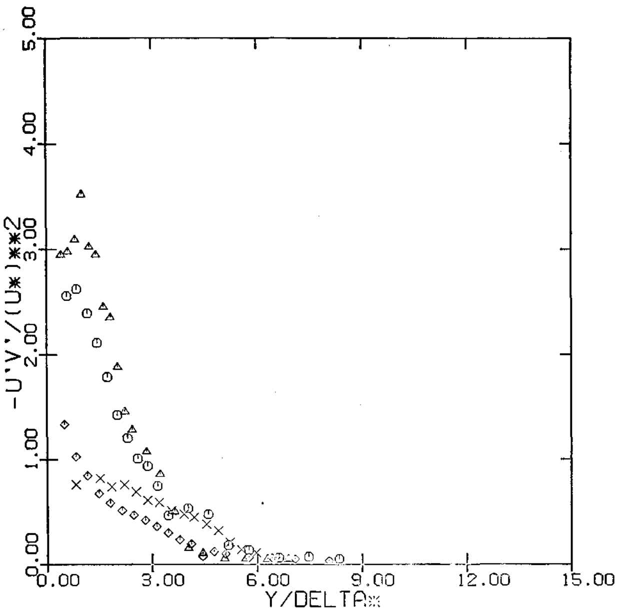


**Fig. 4 Reynolds stress profiles at $Re_L = 4.96 \times 10^6$. Perforated titanium wall: ○, $V_0^+ = 0.14$; Δ, $V_0^+ = 0.19$; ◇, $V_0^+ = 0.00$. Smooth, solid wall (Ref. 9): ×.**

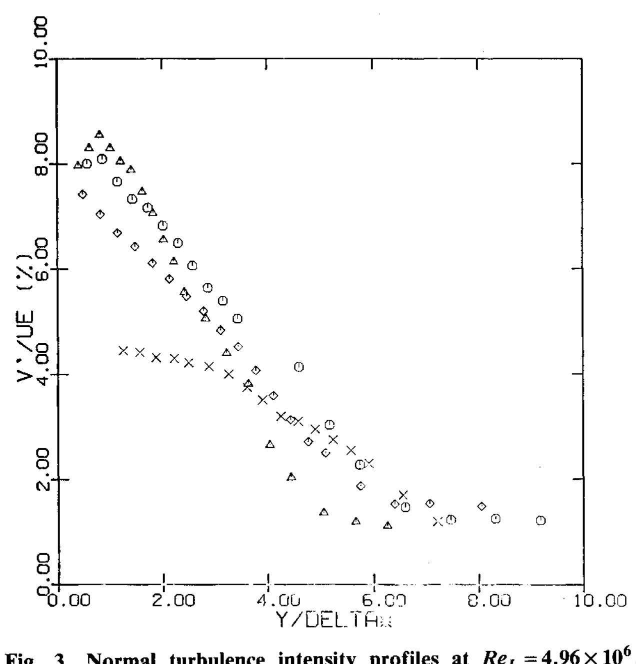


**Fig. 3 Normal turbulence intensity profiles at $Re_L = 4.96 \times 10^6$. Perforated titanium wall: ○, $V_0^+ = 0.14$; Δ, $V_0^+ = 0.19$; ◇, $V_0^+ = 0.00$. Smooth, solid wall (Ref. 9): ×.**

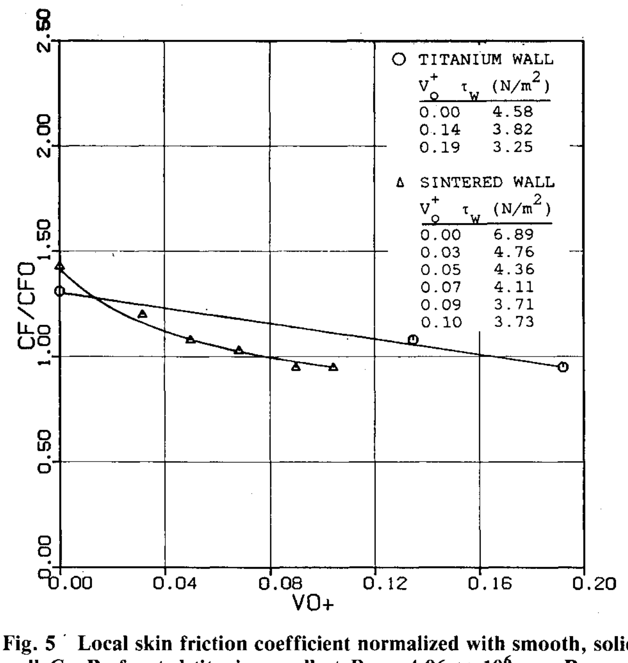


**Fig. 5 Local skin friction coefficient normalized with smooth, solid wall $C_f$. Perforated titanium wall at $Re_L = 4.96 \times 10^6$: ○. Porous, sintered wall at $Re_L = 5.76 \times 10^6$: Δ.**

turbulence to approach the freestream value at a faster rate when compared to the smooth, solid wall.

The normal turbulence intensity results are shown in Fig. 3. The normal turbulence was observed to have a higher value near the wall in comparison to a smooth, solid wall. The normal turbulence also approached the freestream value faster with increasing injection velocity.

The Reynolds stress profiles are presented in Fig. 4. The no injection case for the perforated titanium wall shows a larger Reynolds stress near the wall than does the smooth, solid wall. Injection causes a much larger Reynolds stress, and this is seen to increase with increasing injection velocity. The Reynolds stress for the perforated titanium wall is observed to approach the freestream value in a manner similar to that of the axial and normal turbulence; however, this behavior is not as pronounced.

The local skin friction coefficients and corresponding wall shear data for the perforated titanium wall with and without injection along with previous results for a sintered metal surface in the same apparatus are shown in Fig. 5. Comparison of the results for the no injection case show that the perforated titanium wall has about a 30% higher skin friction coefficient than does the smooth, solid wall. The porous, sintered wall was observed to have a 43% higher skin friction coefficient. The results show that for a given $V_0^+$, the sintered material is more effective in the reduction of skin friction coefficient. A reduction of 33% in $C_f$ is achieved at $V_0^+=0.10$ for the sintered material relative to the no blowing case. In comparison, only a 28% reduction in skin friction coefficient is achieved at higher $V_0^+=0.19$ for the perforated titanium wall. Since the porous, sintered wall is more uniformly porous on the small scale, these results are understandable. The perforated wall produced an array of tiny air jets. Lastly, considerable blowing was required to reduce the $C_f$ of both porous walls below the value of the smooth, solid wall.

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# Momentum/Heat-Transfer Analogy for Turbulent Boundary Layers in Mild Pressure Gradients


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## Introduction

THE heat-transfer rate can be estimated from the skin friction using momentum/heat-transfer analogies without actually solving the energy balance equation. The analogies have been widely adopted for the evaluation of local heat (mass)-transfer rates in the turbulent boundary layers developed even on rotating surfaces.[1,2]

Various analysts[3-5] proposed different formulas on the basis of the Couette flow approximation. However, Spalding[6] employed the von Mises transformation to retain the advection terms. None of these analyses takes account of the pressure gradient effects.

There are only a few methods proposed for nonzero pressure gradients. Reshotko and Tucker[7] developed the method for nonzero pressure gradients in which the Reynolds analogy factor depends on the velocity shape factor alone. Cohen[8] used the integral momentum and energy equations with a number of approximations and obtained the Reynolds analogy factor for compressible flows under nonzero pressure gradients, but the analogy factor is found to become independent of the pressure gradients as the Mach number approaches to zero. Tetervin[9] approximated the distributions of the shear stress and heat flux by third-order polynomials and computed the Reynolds analogy factor for the Prandtl number unity as a function of the velocity shape factor and a Clauser-type pressure gradient parameter. He simply multiplied the resulting analogy factor by Colburn's Reynolds analogy factor so as to take account of the Prandtl number effects. This treatment for the Prandtl number effects seems to lack a sound basis. Because of the limitations of the methods cited above, a different approach seems desirable for the evaluation of the Reynolds analogy factor.

This Note introduces the velocity law of the wall valid for mild pressure gradients and couples it to the temperature law of the wall to obtain the analogy factor as influenced by the pressure gradients and laminar Prandtl numbers (the laminar Prandtl number is varied while the turbulent Prandtl number is assumed to be unity). It will be shown that the resulting general formula for the momentum/heat-transfer analogy is valid for all laminar Prandtl numbers except those characteristic of liquid metals.

## Analysis

The mixing length hypothesis under the linear scale variation leads to

$$\tau=\rho\left(\kappa y\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2 \tag{1}$$

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